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ANALYSIS OF PILED-RAFT FOUNDATIONS SUBJECTED TO GENERAL LOADING

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ABSTRACT

This paper submits a proposed method of analyzing a piled-raft foundation subjected to general loading as the requirements of design for rafts enhanced with piles. In this method, the analysis considers a raft in contact with the ground soil and it can be used for the problems of the variation of soil modulus from layer to layer. Moreover, this method has the advantage that the data is easy to prepare and does not involve creating large meshes as would be required for finite element solutions. Regardless to other factors, the analysis shows that the stiffness ratios (E_{pile}/E_{soil} and E_{raft}/E_{soil}) and pile spacing (S/D) play significant effects on the piled-raft displacements and loadings. To verify the reliability of this method, the obtained results of FLAC-3D are compared with the field tests of a large scale group tested under horizontal loading. Good agreement between field loading and the proposed method is demonstrated.

KEYWORDS: Foundations, Piled-Raft, Interface loads, Soil-Structure Interaction, Displacements, Stiffness Ratio, General Loading

1. INTRODUCTION

It is generally known that the piles play an important role in settlement and differential settlement reduction. Therefore, when the performance of the raft foundation alone does not satisfy the design requirements, piled-raft foundations provide the suitable and aneconomical foundation options for circumstances. Where, the addition of a limited number of piles may improve the ultimate load capacity, the settlements and the thickness of the raft.

Many researchers such as Davis and Poulos [1], Hain and Lee [2], Franke et al [3], Ruesta and Townsend [4], and Poulos [5] are interested to study piled raft foundations. They have focused the efforts towards piled raft foundations and have submitted considerable information about the methods of analysis and design. Randolph [6], De Sanctis et al [7] and Viggiani [8] and others have defined three different design philosophies with respect to piled-rafts as: (a) the conventional approach, where, the piles are designed as a group to carry the major part of the load and some allowance for the contribution of the raft, primarily to ultimate load capacity, (b) creep piling, where, the piles are designed to operate at a working load at which significant creep starts to occur, typically 70-80% of the ultimate load capacity. Sufficient piles are included to reduce the net contact pressure between the raft and the soil to below the pre-consolidation pressure of the soil, and (c) differential settlement control, where, the piles are located strategically in order to reduce the differential settlements, rather than to substantially reduce the overall average settlement.

A several methods of analysis are used for predicting piled-rafts behavior, their capabilities and their limitations. Some of methods are useful only for preliminary design or for checking purposes, and others are capable to give detailed performance predictions and detailed design. Accordingly, researchers submitted several methods of analyzing piled rafts foundations. The main methods are: (a) simplified method [7&9], (b) employing a strip on spring's approach method [5&10], (c) employing a plate on springs approach [7&11], (d) boundary element method [10&12], (e) combining boundary element for the piles and finite element analysis for the raft [2&13], and (f) finite element analyses [14&15]. Where, these methods deal only with piled raft foundations subjected to vertical loading or moments, but not horizontal loads. In this paper, a proposed method of analyzing a piled raft subjected to general loads (vertical loads, horizontal loads and moments).

2. PROPOSED METHOD ANALYSIS

The raft foundation can be subjected to horizontal and vertical loads as well as moments. In this study, the raft is supported by both the soil and the piles and the piled-raft movements and rotations are suggested in three directions (x, y, and z). This analysis indicate that the piled-raft foundations may be separated into the isolated raft which is subjected to external loading {Q} and interface forces $\{P_r\}$, as shown in Figure 1-(a). While,the pile group embedded in a layered soil, is subjected to interfaces forces $\{P_p\}$, Figure 1-(b). The forces between the piles and layered soil can be treated as a series of ring loads applied to nodes along the pile shaft. These loads are both horizontal and vertical. These forces can approximate the continuous forces that act along the pile shaft reasonably well if enough are used.

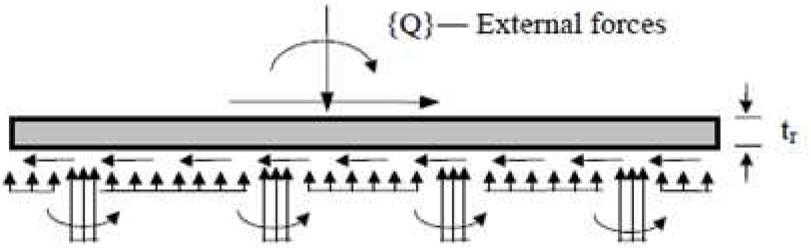
Poulos [5] and Dang et al [14] mentioned that the behavior of a piled-raft foundation is influenced by the interactions between the piles, raft and soil, and consequently interaction factors have been widely adopted for the prediction of the response of a piled-raft. Accordingly for the proposed analysis, the raft is divided into a series of rectangular elements with each pile head assumed to fit within one of the raft elements. The raft is modeled as a thin plate and each element has four nodes and twenty-four degrees of freedom. The interface force applied to any of the raft elements is assumed to be a uniform load over the element. In the other side, the piles and the soil are subjected to interface forces transferred from the raft. The forces acting on the pile heads are assumed to be concentrated loadings and the forces applied to the soil surface are taken to be a series of rectangular blocks of uniform pressure. The displacements of the layered soil and pile heads can be computed.

2.1 Analysis of Raft

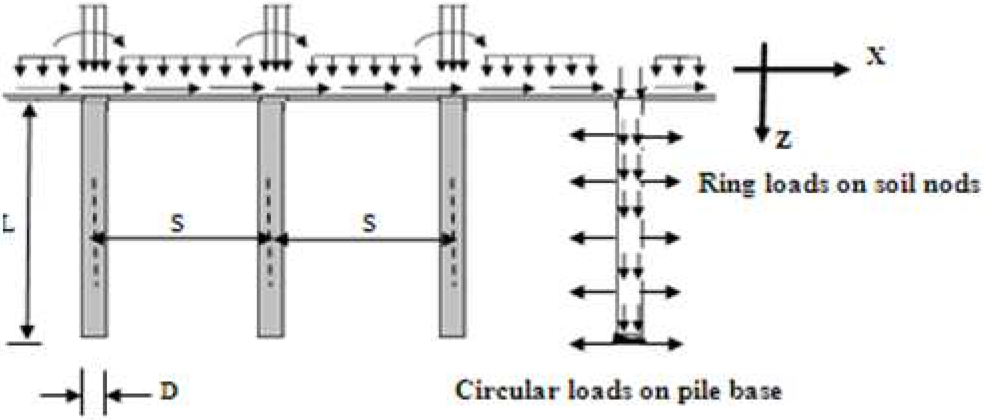
Some nodes on the raft must be restrained from undergoing free body rotations and translations for the analysis of the raft. Two corner nodes of the raft were chosen as points of restraint. Where, the first one is completely fixed in all directions (i.e. six freedoms) and the other is fixed only in the y direction to resist rotation of the raft about the z-axis. The rigid body translations and rotations about the first pinned node of the raft are assumed to be D_x, D_y, D_z, θ_x, θ_y and θ_z. Therefore, the actual displacement (Δ_r) at the center of each raft element may be expressed as:

$$\{\Delta_r\} = [I_r]\{P_r\} + \{a\}D_x + \{b\}D_y + \{c\}D_z + \{d\}\theta_x + \{e\}\theta_y + \{f\}\theta_z + \{\Delta_{ro}\} \tag{1}$$

Where, $[I_r]$ is the influence matrix of the pinned raft, $\{P_r\}$ is the vector of interface loads and moments on the raft elements, $\{\Delta_{ro}\}$ is the vector of displacements at the centers of the raft elements due to applied loads on the pinned raft, and {a}to {f}are auxiliary vectors related to the raft geometry.



(a) Interface Forces on the Raft{P_r}



(b) Interface Forces on the Pile Heads and the Soil Surface {P_p}

Figure 1: Piled-Raft with External Forces and Interface Forces in all Directions

2.2 Analysis of Pile Group

Ta and Small [15] and Small and Zhang [16] stated that the interactions of soil-to-soil, soil-to-pile, pile-to-pile and pile-to-soil must be taken into account to analyze a pile group embedded in a layered soil. The interaction between soil and soil may be directly solved by the finite layer method, while, the other interactions may be obtained by using the method combined with the finite layer method. The displacements at the top of each pile and the centre of each soil surface element under the interface forces transferred from the raft can be expressed as:

$$\{\Delta_p\} = [I_p]\{P_p\} \tag{2}$$

Where,[I_p]is influence matrix of the pile enhanced soil continuum, {P_p}isthe interface load vector between the raft and the pile-enhanced soil, and {Δ_p} is the vector of interface displacement between the raft and the pile-enhanced soil.

2.3 Analysis of Piled-Raft

By considering the compatibility of displacements and the equilibrium of interaction forces between the raft and the soil surface in addition pile heads, it can be obtain that:

$$\{\Delta_r\} = \{\Delta_p\} \tag{3}$$

$$\{P_r\} = \{P_p\} \tag{4}$$

The combination of equations (1) to (4) leads to the follows:

$$([I_p]-[I_r])\{P_p\}-\{a\}D_x-\{b\}D_y-\{c\}D_z-\{d\}\theta_x-\{e\}\theta_y-\{f\}\theta_z=\{\Delta_{ro}\} \tag{5}$$

Then, the equilibrium of applied forces and interface forces acting on the raft gives the following:

$$\{a^{'}\}\{P_p\}=P_x \tag{6}$$

$$\{b^{'}\}\{P_p\}=P_y \tag{7}$$

$$\{c^{'}\}\{P_p\}=P_z \tag{8}$$

$$\{d^{'}\}\{P_p\}=M_x \tag{9}$$

$$\{e^{'}\}\{P_p\}=M_y \tag{10}$$

$$\{f^{'}\}\{P_p\}=M_z \tag{11}$$

Where,(a) P_x, P_y, and P_z are the total loads applied to the raft in the x, y and z directions respectively; (b) M_x and M_y are the total moments applied to the raft about the pinned point, while, M_z is the total moment about z-axis due to P_x and P_y; and (c) {a'}to {f'}are auxiliary vectors related to vectors {a}to {f}.

By solving equations (5) to (11), the interface pressures on the pile-enhanced soil can be estimated. Then, solutions for the displacements in the raft may be obtained by substituting these pressures into Eq. (1).Hence, finite-element method (FEM) for piled-raft foundation is simulated using the commercial program FLAC-3D as the programming language under FLAC package. Where, the simulation was based on FLAC-3D toolboxes which are suitable for equations.

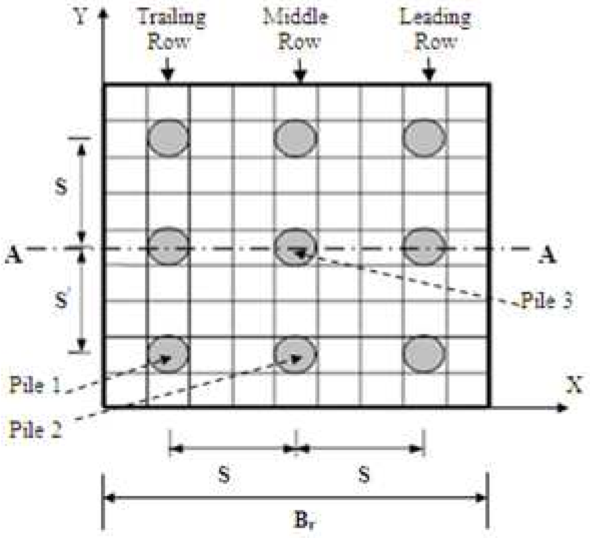
In the FEM, the soil is considered as a hardening model of the case of hyperbolic relationship and the piles are represented by embedded pile elements and can be placed arbitrarily in a soil volume element. The position of the element nodes are created in the soil volume element from the element shape functions. Then, the special interface forms a connection between the pile element nodes and these virtual nodes, and thus with all nodes of the soil volume element. The interaction with soil at the pile skin and at the pile foot is described by means of embedded interface elements. These interface elements are based on 3-node line element with a pairs of nodes instead of single nodes. One node of each pair belongs to the beam element, whereas the other (virtual) node is a point in the 15-node wedge element belonging to soil element. The skin interaction is taken into account by the development of skin traction and the foot interaction is considered by the development of the foot force. So, the computed interface forces of raft and piles can be obtained and compared with the actual forces and applied forces. It is noted that, if the difference between the computed forces and the actual forces is more than 1%, the procedure is repeated to reach the required accuracy.

3 APPLICATION OF THE PROPOSED METHOD

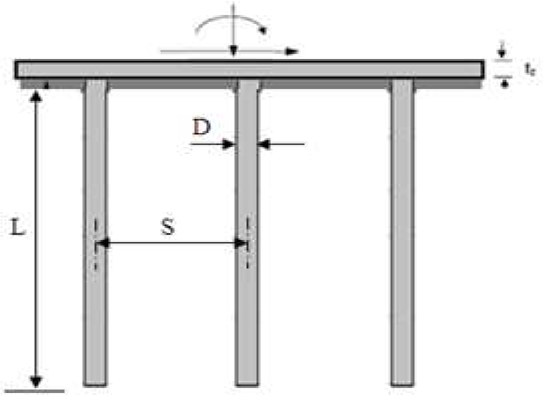
The analysis approach of this proposed method is applied on a square piled-raft foundation with 9 (3×3) piles embedded in a deep uniform soil, as indicated in Figure 2.Poisson’s ratios of the raft and soil are chosen to be 0.15 and 0.35 respectively. The raft thickness (t_r) and the diameters of the piles (D) are taken as 0.6 m (i.e.$t_r/D=1.0$), also, pile group with a spacing ratio between piles (S/D) is 4.0. Moreover, $E_r/E_s=E_p/E_s=2000$, where, E_r, E_p and E_s are the elasticity modulus of raft, pile and soil respectively. The horizontal (Δ_x) and vertical (Δ_y) displacements at the top ofpiles 1, 2 and 3are plotted in Figures 3 and 4 respectively. According to indicated displacements due to the effect of vertical and horizontal loading, it is generally evident that:

- The vertical displacements are less than the horizontal displacements by about 45%.
- The displacements at corner pile (Pile 1) are slightly more than that at others. But, the displacements increase with the increase of applied horizontal and vertical loading.

Unquestionably, in the design of piled-raft foundations, obtaining the distribution of the bending moment and its maximum value in the raft is very important. Based on moment values, the quantity and distribution of reinforcing steel used for the raft are determined. Figure 5 shows the distribution of total bending moment variation along section A-A of the raft, as shown in Figure 2 (piled-raft with nine), at the total load of about 25,000 kN. The results of moment distribution indicated that the maximum bending moment is distributed in the central region of the raft and the moment reduces from the center to the edges of the raft. Furthermore, the same trend of moment distribution was stated by using the commercial structure analysis program such as SAP-2000 and PLAXIS-3D.



(a) Plan



(b) Cross-Section A-A

Figure 2: Schematic Diagram of Pile Group

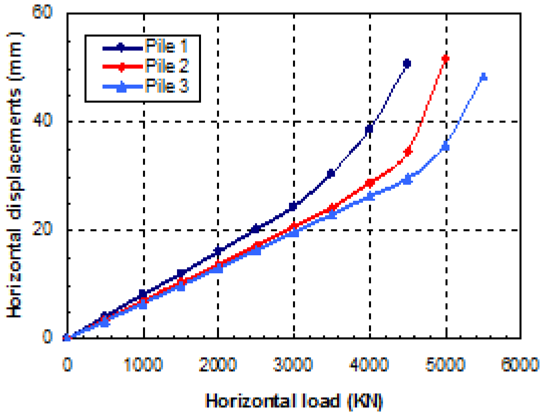


Figure 3: Horizontal Displacements at the Top of Pile

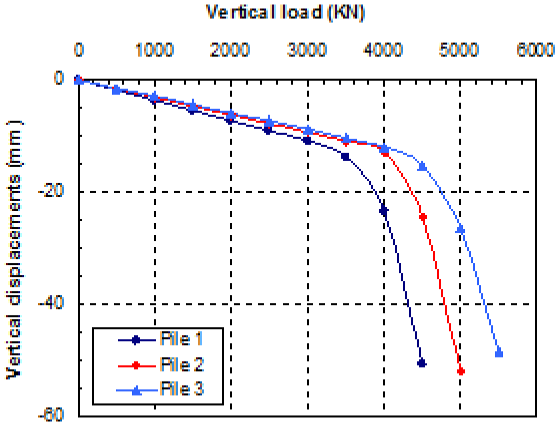


Figure 4: Vertical Displacements at the Top of Pile

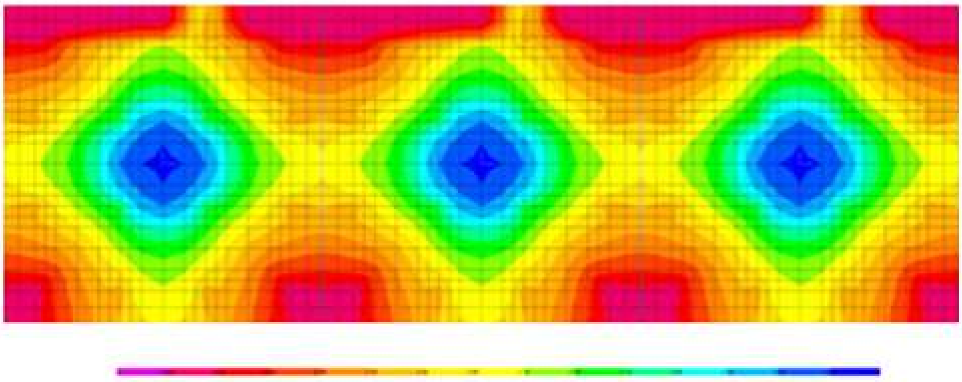


Figure 5: Distribution of Bending Moment Variation Along Section A-A

On the other side, for both the horizontal and vertical loading, normalized horizontal (Δ_{xx}) and vertical displacements (Δ_{zz}) are:

$$\Delta_{xx} = \frac{E_s D}{q_x B_r L_r} . \Delta_x \qquad (12)$$

$$\Delta_{zz} = \frac{E_s D}{q_z B_r L_r} . \Delta_z \tag{13}$$

Where, (a) Δ_x and Δ_z are respectively the actual horizontal and vertical displacements, (b) q_x and q_z are respectively the uniform horizontal and vertical loads, and (c) $B_r$and $L_r$are respectively the breadth and length of the raft.

In this study, the effect of the studied variables on the pile raft foundation behavior is presented in dimensionless form, as shown in Figures 6-12. These figures show the relationships between normalized displacements and loading percent carried by pilesagainst pile-soil stiffness ratios (E_p/E_s), raft-soil stiffness ratios (E_r/E_s), and pile spacing ratios (S/D) as the variation of vertical displacement, horizontal displacement, vertical loading and horizontal loading carried by piles. According to results of analysis and the relationships, as plotted in Figures 6-12, the studied variables affecting the piled-raft foundation can be described as the followings:

3.1 Effect of Pile-Soil Stiffness Ratio on Displacement and Load Distribution

According to the input data as the pile spacing ratio (S/D)is 4, the soil modulus (E_s)is 10 MPa, raft-soil stiffness ratio (Er/Es)is2000, the pile slenderness ratio (L/D)is 25 and the soil depth was assumed to be infinite. The results of the analysis are shown in Figures 6 to 9 for different pile-soil stiffness ratios ($R=E_p/E_s$). For the case of a uniform applied shear loading ($\tau_{app.}$) to the raft in the x-direction, interface shear stress ($\tau_{int.}$) along section A-A at the pile heads and soil surface is represented as a ratio with applied shear loading (shear ratio=$\tau_{int.}/\tau_{app}$). The variations of $\tau_{int.}$ along section A-A are shown in Figure 6 and the effect of E_p/E_s ratios on the displacements and pile loading is plotted in Figures 7 and 8. Due to the analysis results, it is noted that:

- The highest values of $\tau_{int.}$ occur at the pileheads and at high values of pile-soil stiffness ratios (E_p/E_s), while, $\tau_{int.}$ values are decreased on the soil surface. For instant, at the coordinate points (x/B_r) of 0.16 and 0.33 for R (E_p/E_s) =1000, the values of shear ratio ($\tau_{int.}/\tau_{app.}$) are 9.7 and 0.18 respectively. So, the value of shear ratio at the pile heads is extremely increased that at the soil surface.
- The increase of the pile-soil stiffness ratio (Ep/Es) leads to a reduction in the horizontal displacement of the piled raft under horizontal load, see Figure 6. For example, the values of normalized displacements (Δ_{xx}) for horizontal displacements are 0.038 and 0.023at E_p/E_s ratios of 10 and 1000 respectively.
- The reduction in vertical displacement (Δ_z) also occurs for the piled raft under vertical loading. However, when E_p/E_s ratio is low, Δ_z reduces fairly rapidly. But, when E_p/E_s ratio is in excess of 1000,Δ_z of the piled raft is not very sensitive to E_p/E_s ratio.
- The horizontal load percentage carried by the piles increases as E_p/E_s ratio increases. While, the vertical load carried by the piles is slightly increasing when E_p/E_s ratio exceeds than 1000.

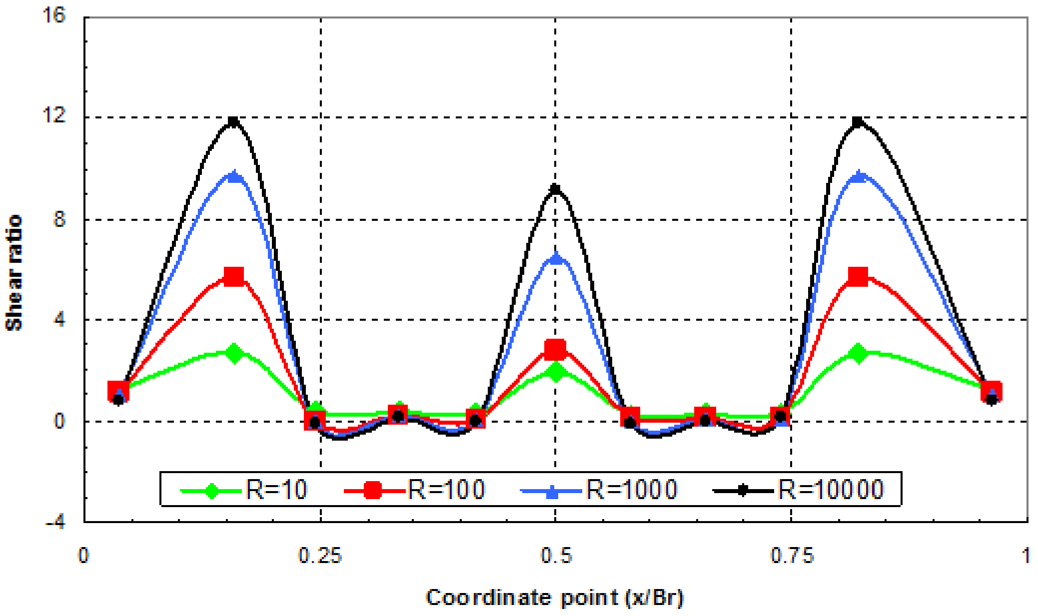


Figure 6: Interface Shear Stress Variation Along Section A-A

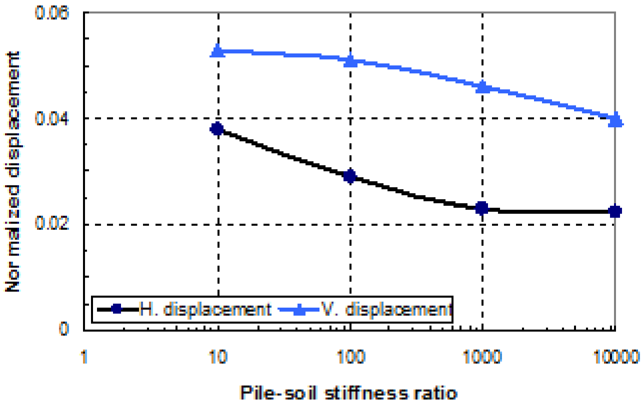


Figure 7: Effect of E_p/E_s Ratios on Piled-Raft Displacements

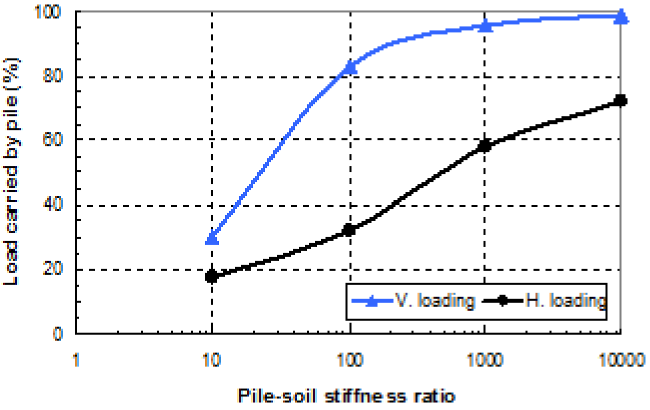


Figure 8: Effect of E_p/E_s Ratios on Pile Loading

3.2 Effect of Raft-Soil Stiffness Ratio on Displacement and Load Distribution

The chosen input data to study the effect of the raft-soil stiffness ratio (E_r/E_s) as: (a) the pile spacing ratio (S/D) is

4, (b) the pile slenderness ratio (L/D) is 25, (c) the stiffness ratio of pile-soil (E_p/E_s)is 2000, and (d) the soil modulus (E_s) is 10 MPa. The effect of raft-soil stiffness ratios (E_r/E_s) on the displacements and pile loading is indicated in Figures 9 and 10. According to the analysis results, it is evident that:

- The raft-soil stiffness ratio (E_r/E_s) has a limited influence on the displacement of the piled-raft whether it is subjected to horizontal loading or vertical loading, Figure 9. For instant, at E_r/E_s ratios of 10 and 1000, the values of normalized displacements (Δ_{xx}) for horizontal displacements are 0.0215 and 0.0218 respectively; also, the values of Δ_{zz} are 0.047 and 0.045 respectively. This is not similar the effect of pile-soil stiffness that has a large effect on displacements, see Figure 7.
- The increase of E_r/E_s ratio will give an obvious increase in the loading percentage carried by piles. For higher raft-soil stiffness ratios (i.e. greater than 100) the variation of the raft-soil stiffness ratio will have only a small effect on the loading distribution. Where, at E_r/E_s values of 10, 100 and 1000, the percentages of horizontal load carried by piles are 81%, 96% and 98% respectively and the percentages of vertical load carried by piles are 41.5%, 58% and 63% respectively.

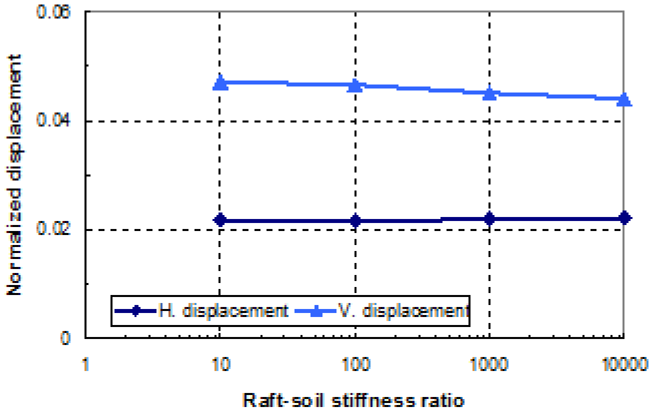


Figure 9: Effect of E_r/E_s Ratios on Piled Raft Displacements

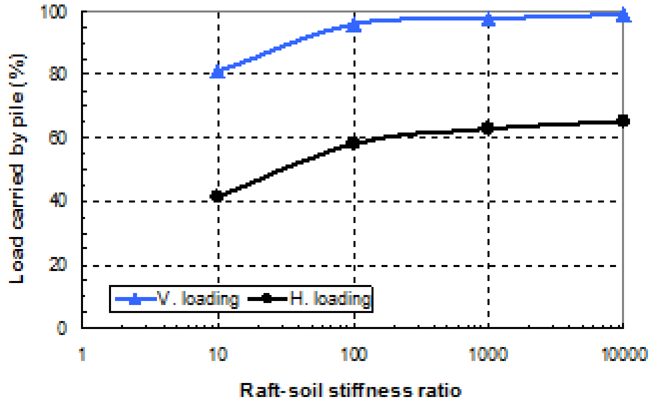


Figure 10: Effect of E_r/E_s Ratios on Pile Loading

3.3 Effect of Pile Spacing Ratio on Displacement and Load Distribution

Unquestionably, to study the effect of pile spacing ratio (S/D) on the displacements and loading distributions of piled-raft foundation, the input data must be similar to aforementioned data, where, L/D=25, Es=10 MPa, and Ep/Es=Er/Es=2000. The computed results about the effect of S/D ratios on the displacements and pile loading is shown in Figures 11 and 12.The analysis results indicated that:

- The pile spacing ratio has a pronounced effect on the displacement of the piled-raft foundation whether the piled raft is subjected to horizontal loading or vertical loading especially for small pile spacing ratios. For instant, at S/D ratios of 3 and 6, the values of normalized displacements (Δ_{xx}) for horizontal displacements are 0.06 and 0.039 respectively; also, the values of Δ_{zz} are 0.029 and 0.02 respectively.
- The increase in pile spacing ratio leads to significant reduction of the horizontal loading carried by the piles, Figure 12.Where, at S/D values of 3 and 8, the percentages of horizontal load carried by piles are 75% and 48% respectively.
- The pile spacing ratio has a small influence on the percentage of the vertical loading carried by the piles. For example, at S/D values of 3 and 6, the percentages of vertical load carried by piles are 99% and 94% respectively.

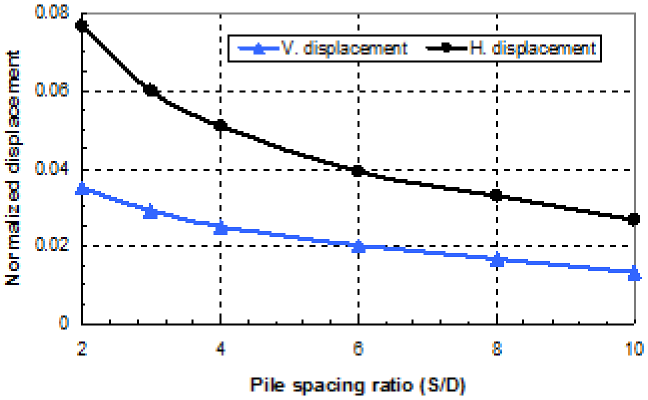


Figure 11: Effect of S/D Ratios on Piled Raft Displacements

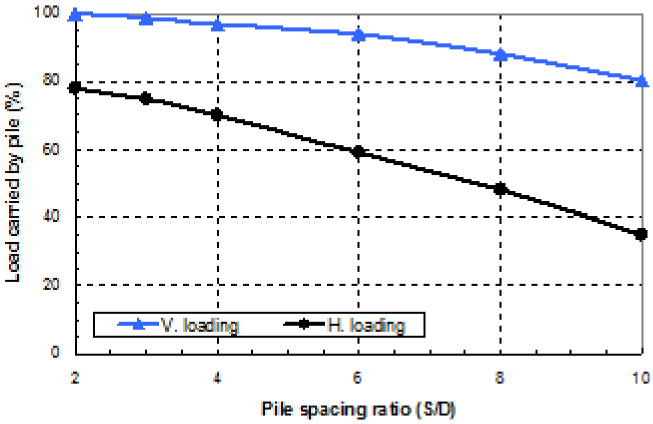


Figure 12: Effect of S/D Ratios on Pile Loading

4. COMOARISON WITH LARGE-SCALETEST RESULTS

The computed results of present study are compared with the results of field tests carried out by Small and Zhang

[4&11]. Where, an isolated single pile and a large-scale group with a spacing ratio of 3 were tested under horizontal loading. The pile cross-section consisted of a 350 mm diameter steel pipe (9.5 mm thick) embedded in concrete such that the pile diameter was 0.75 m. The ratio of the embedded pile length to the pile diameter is 18.42. The modules of elasticity of the concrete and the steel pipe were given as 34,500 MPa and 190,300 MPa, respectively. From the equation $E_c.I_c + E_{pipe}.I_{pipe} = E_{eq.}I_{eq,}$ the equivalent modulus of the piles may be as 43,600 MPa. The Poisson's ratio (ν) of the soil was taken as 0.35. According to field dilatometer and pressure-meter tests, the modulus of the soil layer slightly increases with depth and as it has less effect on the pile behavior. So, the soil layer is assumed to have a homogeneous modulus. According to the results of the in-situ dilatometer and pressure-meter tests, the variation of the soil modulus with depth was approximately determined and varies from 15.35 MPa at the ground surface to 21.35 MPa at a depth of 6 meters.

The moment in the pile predicted by the present method is compared with the measured moment. The measured average moment in the piles located in each row has been plotted in Figures 13 and 14 separately. It should be noted that the average moment in the leading row of piles is exactly the same as that of the trailing row in theory when the same loading is applied to each pile in the group. Therefore, there is only one bending moment curve for both leading and trailing rows, and one curve for the middle leading row and middle trailing row, Figures 13 and 14. The results demonstrate that:

- The predicted moments are in good agreement with the measured moments for each row of piles.
- The difference between the predicted maximum moment and measured maximum moment is about 11.4% for the leading and trailing rows and only about 7.2% for the middle row. But, many of the predicted moment and measured moment are approximately equals.

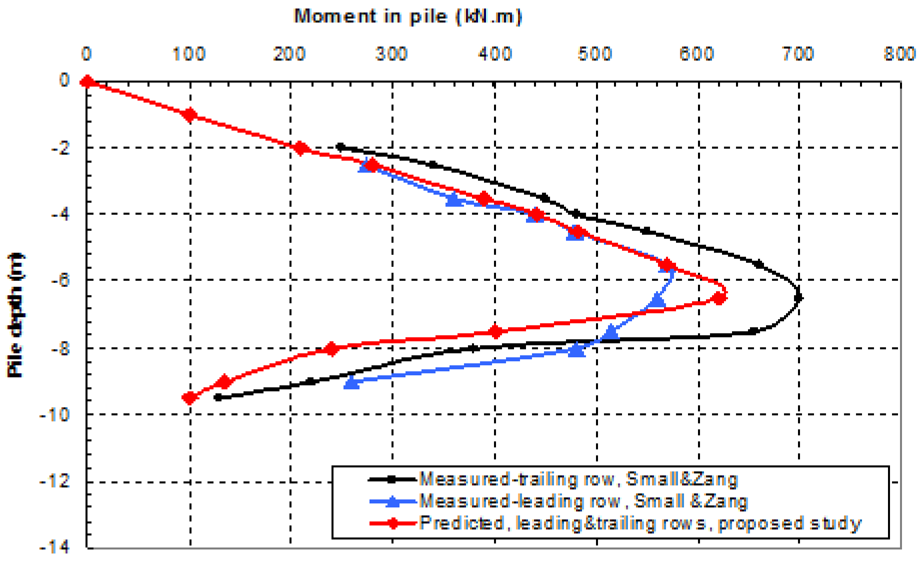


Figure 13: Measured and Predicted Moments in Piles of Leading and Trailing Rows

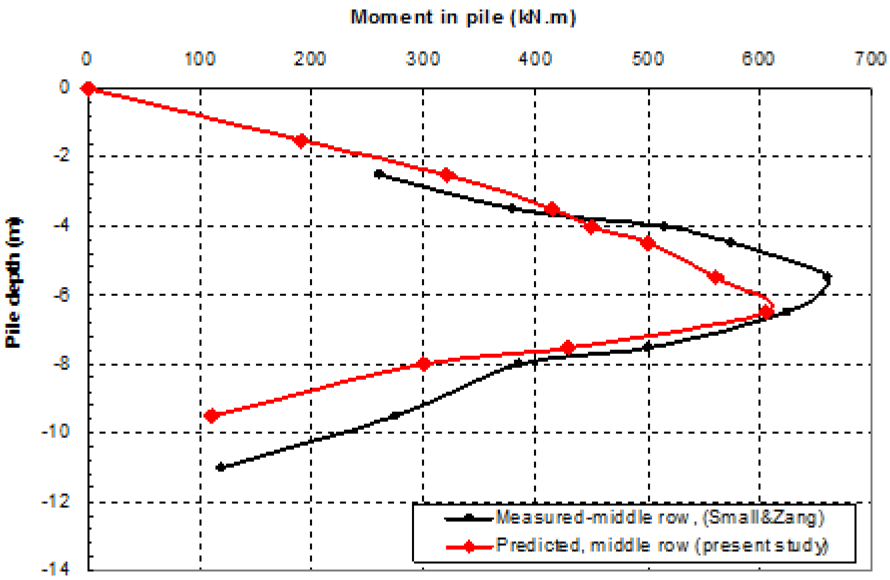


Figure 14: Measured and Predicted Moments in Piles of Middle Row

5. CONCLUSIONS

A method for the analysis of piled-raft foundations using three-dimensional finite element (FLAC-3D) is submitted in this paper. The results of piled-raft displacements and the load sharing between the piles and the raft are presented. Based on the analysis results, the following conclusions could be drawn:

- The highest values of interface shear stress occur at the pile heads and at high values of pile-soil stiffness ratios (E_p/E_s), while, lower values are on the soil surface at small values of E_p/E_s ratio.
- As the increase of the pile-soil stiffness ratio (Ep/Es), the displacement ofthe piled-raft under horizontal and vertical loading. But, at E_p/E_s ratio is in excess of 1000, horizontal displacement (Δ_z) is not very sensitive.
- The vertical and horizontal load percentage carried by the piles increases as pile-soil stiffness ratio (E_p/E_s) increases.
- Under horizontal or vertical loading, the raft-soil stiffness ratio (E_r/E_s) has a limited influence on the displacements of the piled-raft foundations.
- The increase of E_r/E_s ratio leads to an obvious increase in the loading percentage carried by piles.
- Under horizontal or vertical loading, the pile spacing ratio (S/D) has a pronounced effect on the displacement of the piled-raft foundation especially at small pile spacing ratios.
- As the increase in pile spacing ratio(S/D) the significant reduction of the horizontal loading carried by the piles. Also, the pile spacing ratio has a small influence on the percentage of the vertical loading carried by the piles.
- The proposed method can estimate the settlement of a piled raft foundation quite accurately and the distribution of stress in the raft reasonably well. Also, it allows for the use of any structural commercial program to solve the piled raft foundation problem. In addition to that, the piled-raft foundation problem can

be solved effectively through a combination of structural responses and geotechnical characteristics without a complex model of the soil and foundation.

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